Modeling K-Terminal Network Reliability Under Uncertainty Using Fuzzy Arithmetic

Amarjeet Rajora, Dr. Anurag Singh Baghel, Isha Singh

Department of Computer Science and Engineering
Gautam Buddha University, Greater Noida, India
Corresponding Author: asb@gbu.ac.in

ABSTRACT

The problem of assessing reliability when there is uncertainty in the available information on network failures constitutes a serious limitation of classical probabilistic approaches to reliability estimation. Fuzzy set theory provides an attractive solution through the use of fuzzy numbers to represent the reliability of the components involved, thereby providing a mathematical foundation for dealing with epistemic uncertainty. Although several studies have investigated fuzzy reliability models for series-parallel and bridge networks, the issue of fuzzy K-terminal reliability, which involves the connection of only a selected set of nodes, has not yet received substantial attention in the context of fuzzy sets. Each edge in the network is modeled using trapezoidal fuzzy probabilities defined by four-parameter membership functions to capture uncertainty. The analysis incorporates minimal Steiner tree concepts for representing feasible connectivity among terminal nodes, while system reliability is evaluated using an inclusion-exclusion framework under fuzzy arithmetic operations. α-cut representation is employed to characterize and reconstruct the resulting fuzzy reliability structure, and Monte Carlo simulation is used as an external validation mechanism. The methodology is implemented computationally in Python and supported through graphical analysis for interpretability. The final results indicate that the fuzzy K-terminal reliability has a defined support and core, showing high but not near-certain connectivity among the designated terminal nodes. Overall, the proposed framework demonstrates the effectiveness of fuzzy arithmetic in assessing K-terminal network reliability under epistemic uncertainty.

Keywords: Fuzzy reliability, K-terminal network, Steiner tree, trapezoidal membership function, α-cut analysis, Inclusion-Exclusion, Monte Carlo simulation, non-series parallel network.

I. INTRODUCTION

The reliable operation of interconnected networks such as telecommunications backbones, power grids, transportation systems, and computer networks is a subject of enduring importance in systems engineering. The classical theory of reliability describes the behavior of components with exact probability values, which assumes availability of adequate historical failure data of consistent quality. Other modeling methods, such as Bayesian network representations of fault trees, have been proposed to deal with dependability problems in systems with structural uncertainty [8]. In real-world applications, on the other hand, failure probabilities are frequently estimated based on inadequate sample sizes, expert opinion, or tests conducted in a non-representative environment. Such a scenario may result in an illusion of precision by crisp probabilistic models and may thus be detrimental to safety or efficiency considerations in engineering designs.

The introduction of fuzzy set theory by Zadeh [1] in 1965 paved the way for dealing with uncertainty using a systematic model that goes beyond probabilistic approaches. Fuzzy sets allow the assignment of a membership function to each uncertain parameter; hence, the vagueness associated with the data can be captured while still enabling quantitative analysis. Fuzzy probability takes the fuzzy approach a step further into the realm of reliability engineering, representing the probability of a component's survivability as a fuzzy number rather than a single scalar value [6]. Chowdhury and Misra showed that such an approach was viable even when applied to a non-series parallel bridge network, with multiplication and complementation being the key operations [2].

One of the significant types of network reliability questions concerns K-terminal reliability, where the objective is to establish whether a certain subset K of network vertices can be reached from one another, considering that every link may fail individually [3]. K-terminal reliability is more comprehensive and applicable than all-terminal reliability because most practical networks do not require all vertices to be connected; instead, only a select few vertices, such as servers, substations, or relay stations, must be connected. The primary components of K-terminal reliability are minimal Steiner trees - the smallest sets of links that can connect all vertices of *K*.

While the significance of K-terminal reliability is well established alongside the shortcomings of using crisp probabilities in uncertain settings, the field at the interface of these two topics remains sparsely investigated. Fuzzy reliability models have largely been limited to those that exhibit series-parallel composition, for which recursive relations can be easily developed. For non-series parallel topologies with arbitrary terminal sets, however, the explicit identification of Steiner trees and the use of fuzzy

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Inclusion-Exclusion principles will be needed; steps that have not been systematically implemented or validated in the published literature for the K-terminal case [10]. In particular, if minimal Steiner trees share common edges, then their survival events are statistically dependent and cannot be combined through an independence-based product of complements. This dependence issue becomes central in any correct reliability formulation.

This paper attempts to fill this gap by providing a comprehensive computational framework for evaluating fuzzy *K*-terminal reliability. As an example, we consider a network of 5 nodes and 7 edges with terminal nodes *K* = {1, 3, 5}. The reliability of each edge is represented by a trapezoidal fuzzy number as per the definition provided by Chowdhury and Misra [2]. We enumerate all possible minimal Steiner trees and compute the fuzzy system reliability using a corrected Inclusion-Exclusion formulation evaluated at α-cut levels so that shared-edge dependence among Steiner trees is handled explicitly. An α-cut approach is employed to derive the system membership function, and Monte Carlo simulation is additionally used to validate the corrected fuzzy reliability interval. The whole process has been coded in Python programming language to yield numerical and graphical results.

The remainder of this document is organized as follows. Section II defines the research gap and clearly states the objectives of the study. Section III describes the research methodology in detail. Section IV highlights the results of the experiments conducted. Section V is the conclusion of the paper.

II. RESEARCH GAP AND OBJECTIVE

Research on fuzzy reliability has increased steadily since the seminal work of Zadeh [1] and its application to fault trees and series-parallel systems [4]. Nevertheless, a review of current literature reveals a number of significant gaps that have led to the development of this research.

First, the majority of research on fuzzy reliability is limited to topologies that admit series-parallel decomposition. The structure of such networks can be simplified through iterative reduction by aggregating subsystems either in series or in parallel, resulting in a reliability formulation that is comparatively easier to formulate. However, in practice, real-world networks rarely conform to this restricted topology of series-parallel structures. Other topologies like bridge networks, mesh networks, and rings used in communication networks need to be analyzed using a completely different approach, yet they remain underrepresented in the fuzzy reliability literature. Even when fuzzy models have been generalized to non-series-parallel systems, the analysis usually has only covered single failure mode situations and has not adequately captured multi-modal failures.

Second, even among studies investigating non-series parallel networks, the focus is predominantly on the case of all terminal reliability. This involves requiring all nodes in the network to remain mutually connected. A much more general concept of K-terminal reliability, where the connectivity requirement is imposed only for a selected few critical nodes, would be much more relevant in practice [7].

Third, while techniques for finding all-pairs shortest paths in crisp graphs like the one developed by Abraham have been proposed [5], there is no comparable algorithm for handling shortest-path computation in fuzzy arithmetic graphs. The lack of a computational algorithm for solving the problem in fuzzy networks is a major drawback in the existing literature.

Fourth, although the α-cut approach has been proven to be an effective method for analyzing fuzzy outcomes, there is no proof in any prior research that demonstrates how to utilize this method systematically to validate and reconstruct fuzzy K-terminal reliability over non-series parallel networks.

A. Research Objectives

Motivated by the gaps identified above, the goals of this study are:

- To extend the fuzzy reliability approach of Chowdhury and Misra [2] from all-terminal bridge network analysis, to analysis of K-terminal reliability in a non-series parallel network of 5 nodes and 7 edges.
- To implement a Steiner tree enumeration algorithm in which all minimal subsets of edges linking a given set of terminals, K, are exhaustively searched for via breadth-first search.
- To realize an entire fuzzy arithmetic pipeline for the computation of system K-terminal reliability through a corrected Inclusion-Exclusion formulation in which dependence among minimal Steiner trees is explicitly handled rather than beign approximated through independence as in the base paper [2].
- To conduct α-cut based analysis of the entire membership set in order to validate the trapezoidal membership function of the obtained system reliability.
- To validate the corrected fuzzy reliability interval using Monte Carlo simulation [12] with α-cut sampling and BFS-based connectivity testing.
- To demonstrate implementation of complete Python code that can be used to plot validation results.

B. Novelty and Contribution

The primary novelty of this study is the systematic integration of three well-established methods in one system of analysis for evaluation purposes, namely, the algorithm of fuzzy trapezoidal arithmetic, generation of minimal Steiner trees, and α-cut membership estimation technique. In addition, the study corrects the independence assumption implicit in the product-of-complements formula by explicitly modeling the dependence among Steiner trees that share edges, and it

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supplements the analytical method with Monte Carlo simulation [12] as an external validation mechanism.

III. RESEARCH METHODOLOGY

This section presents a complete methodology for assessing the fuzzy K-terminal reliability of a non-series parallel network. It is structured into six sequential stages: Network Definition, Fuzzy Probability Modeling, Steiner Trees Enumeration, Evaluation of Fuzzy Reliability, α-Cut verification, and Monte Carlo validation.

A. Network Topology and Terminal Set Definition

The network under study consists of five nodes and seven undirected edges, forming a non-series parallel topology that cannot be reduced through simple series or parallel combinations. The edges are defined as follows: e_1 connects nodes 1–2, e_2 connects nodes 1–3, e_3 connects nodes 2–4, e_4 connects nodes 3–4, e_5 connects nodes 2–3, e_6 connects nodes 4–5, and e_7 connects nodes 1–5.

The terminal set is designated as $K = \{1, 3, 5\}$, meaning that nodes 1, 3, and 5 must remain mutually reachable for the system to be considered operational. Nodes 2 and 4 serve as intermediate Steiner nodes and are not required to participate in the connectivity requirement, though they may be traversed as part of feasible connecting paths between the terminal nodes.

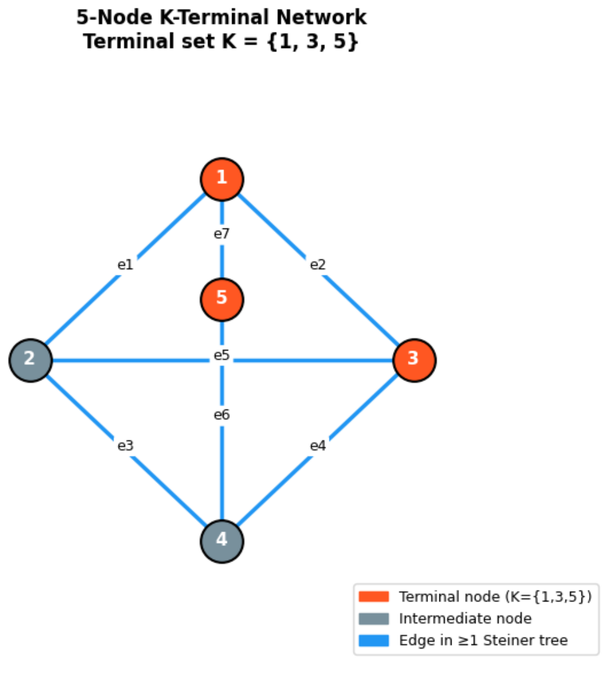

Fig. 1. 5-node, 7-edge network topology with terminal set $K = \{1, 3, 5\}$. Terminal nodes are highlighted, and all edges belonging to at least one minimal Steiner tree are shown.

B. Fuzzy Probability Modeling

Each edge e_i is assigned a trapezoidal fuzzy probability P_i characterized by four parameters $(\alpha_1, \alpha_2, \beta_2, \beta_1)$, where α_1 and β_1 denote the lower and upper bounds of the support, and α_2 and β_2 define the core interval over which the membership function equals unity. The membership function is formally expressed as [2]:

$$\mu_{P_i}(p) = \{0, p < \alpha_{i1}\ \frac{p-\alpha_{i1}}{\alpha_{i2}-\alpha_{i1}}, \alpha_{i1} \le p < \alpha_{i2}\ 1, \alpha_{i2} \le p \le \beta_{i2}\ \frac{\beta_{i1}-p}{\beta_{i1}-\beta_{i2}}, \beta_{i2} < p \le \beta_{i1}\ 0, p > \beta_{i1}$$

The fuzzy probabilities assigned to the seven edges are tabulated in Table I. These values are selected to reflect realistic variation in edge reliability uncertainty, with higher confidence edges exhibiting narrower support widths and lower-confidence edges exhibiting wider spreads.

TABLE I
FUZZY PROBABILITIES OF NETWORK EDGES

Edge	Nodes	α_1	α_2	β_2	β_1
e_1	1-2	0.70	0.80	0.90	0.95
e_2	1-3	0.60	0.75	0.85	0.92
e_3	2-4	0.65	0.78	0.88	0.94
e_4	3-4	0.55	0.70	0.82	0.90
e_5	2-3	0.72	0.82	0.91	0.96
e_6	4-5	0.68	0.79	0.89	0.95
e_7	1-5	0.50	0.65	0.80	0.88

C. Fuzzy Arithmetic Operations

Two fundamental fuzzy arithmetic operations, drawn directly from the base paper framework, are employed throughout the reliability computation [2].

Multiplication: The fuzzy product of two trapezoidal fuzzy numbers Pi and Pj is approximated using the Tanaka et al. [3] interval arithmetic scheme:

$$P_i \cdot P_j = \left(\alpha_{i1} \cdot \alpha_{j1},\ \alpha_{i2} \cdot \alpha_{j2},\ \beta_{i2} \cdot \beta_{j2},\ \beta_{i1} \cdot \beta_{j1}\right)$$

This operation is applied iteratively across all edges within a Steiner tree to compute the tree's joint survival probability.

Complementation: The fuzzy complement of Pi, representing the probability that edge e_i fails, is given by [2]:

$$\underline{P_i} = (1 - \beta_{i1},\ 1 - \beta_{i2},\ 1 - \alpha_{i2},\ 1 - \alpha_{i1})$$

This operation preserves the trapezoidal form and is applied to obtain the failure probability of each Steiner tree as a whole.

D. Minimal Steiner Tree Enumeration

A brute-force enumeration algorithm is employed to identify all minimal Steiner trees connecting the terminal set $K = \{1, 3, 5\}$. The algorithm proceeds as follows:

1. Generate all possible subsets of the edge set $\{e1, e2, ..., e7\}$, ordered by increasing cardinality.

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2. For each subset, construct the induced subgraph and apply breadth-first search (BFS) originating from one terminal node [5].
3. If all terminal nodes in K are reachable within the induced subgraph, test minimality by verifying that no proper subset of the candidate edge set also achieves full terminal connectivity.
4. Retain the subset as a minimal Steiner tree only if both conditions: connectivity and minimality are satisfied.

This procedure is exact for networks of the present scale and yields a complete, duplicate-free collection of minimal Steiner trees. State-space decomposition methods offer an alternative formulation for analyzing stochastic flow networks, though they incur higher computational overhead for dense topologies [9]. Applied to the 5-node network, the algorithm identifies eleven minimal Steiner trees, as listed in Table II.

TABLE II
MINIMAL STEINER TREES FOR $K = \{1, 3, 5\}$

Tree	Edge Set
T_1	$\{e_1, e_3, e_4, e_6\}$
T_2	$\{e_1, e_3, e_4, e_7\}$
T_3	$\{e_4, e_6, e_7\}$
T_4	$\{e_1, e_4, e_5, e_6\}$
T_5	$\{e_2, e_4, e_6\}$
T_6	$\{e_3, e_5, e_6, e_7\}$
T_7	$\{e_2, e_3, e_5, e_6\}$
T_8	$\{e_1, e_2, e_3, e_6\}$
T_9	$\{e_1, e_3, e_5, e_6\}$
T_{10}	$\{e_1, e_5, e_7\}$
T_{11}	$\{e_2, e_7\}$

E. Fuzzy System Reliability via Inclusion-Exclusion

The K-terminal fuzzy system reliability R_s is derived using the principle that the system is operational if and only if at least one minimal Steiner tree survives intact. Let S_m denote the event that Steiner tree T_m survives. The system reliability is therefore given by the union of all Steiner-tree survival events:

$$R_s = P(\cup_{m=1}^{M} S_m)$$

where M = 11 is the total number of minimal Steiner trees. A crucial point is that these Steiner trees are not independent. Many of them share common edges; for example, edge e6 occurs in 8 of the 11 minimal Steiner trees: T_1, T_3, T_4, T_5, T_6, T_7, T_8, and T_9. Similarly, edges e_1 and e_3 each appear in 6 trees, further reinforcing the dependence structure among Steiner trees. Therefore, the survival or failure of one Steiner tree is statistically related to the survival or failure of another whenever the two trees share one or more edges. Because of this shared-edge structure, the product-of-complements formula based on independent tree failures is not mathematically valid for this problem.

Accordingly, the system reliability must be computed using the correct Inclusion-Exclusion principle:

$$R_s = \sum_{i} P(S_i) - \sum_{i<j} P(S_i \cap S_j) + \sum_{i<j<k} P(S_i \cap S_j \cap S_k) - \cdots$$

For any subset of Steiner trees, the corresponding intersection probability is computed over the union of all edges contained in that subset. In other words, if trees T_i and T_j both survive, then every edge in $T_i \cup T_j$ must survive. Hence, shared edges are automatically accounted for through union-based edge sets in the intersection terms.

Since direct fuzzy subtraction in the Inclusion-Exclusion expansion can lead to excessive interval widening, the corrected formulation is evaluated at each α-cut level using crisp arithmetic. For each $\alpha \in [0,1]$, the lower and upper system reliability bounds are computed separately by applying IEP to the lower and upper endpoints of the α-cut intervals of the edge fuzzy probabilities. The fuzzy reliability number is then reconstructed from the resulting α-level bounds as follows:

$$a_1 = L(0), \quad a_2 = L(1), \quad b_2 = U(1), \quad b_1 = U(0)$$

Thus, the final fuzzy system reliability is obtained not from independent tree failures, but from a corrected α-cut Inclusion-Exclusion computation that explicitly preserves dependence among shared-edge Steiner trees.

F. α-Cut Verification

To derive and validate the membership function of Rs, an α-cut scan is conducted in the range $\alpha \in [0,1]$ with uniform discretization at eleven divisions. At each value of α, the lower and upper reliability bounds are computed directly from the corrected Inclusion-Exclusion procedure using the corresponding lower and upper endpoints of the edge α-cut intervals. Thus, rather than being inferred only from a previously assumed trapezoidal output, the α-cut method here serves as the actual computational mechanism through which the fuzzy system reliability is constructed.

The resulting family of intervals [L(α), U(α)] defines the membership structure of Rs and allows the trapezoidal approximation of the final fuzzy number to be reconstructed as:

$$L(\alpha) = a_{1_{R_s}} + \alpha \cdot \left(a_{2_{R_s}} - a_{1_{R_s}}\right)$$
$$U(\alpha) = b_{1_{R_s}} - \alpha \cdot \left(b_{1_{R_s}} - b_{2_{R_s}}\right)$$

where a_1, a_2, b_2, b_1 are obtained from the corrected α-cut Inclusion-Exclusion output. In this way, α-cut analysis is not treated as a stand-alone proof of correctness, but as a reconstruction and interpretive tool for the analytically computed fuzzy reliability interval.

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IV. EXPERIMENTAL RESULTS AND VALIDATION

This section provides an explanation of the findings that were gained using the proposed computation technique. All the outcomes were computed using the Python program and analyzed graphically. The findings are structured in accordance with the four output panels that were provided by the system and a Monte Carlo validation plot.

A. Network Topology

Fig. 1 shows the topology of the network consisting of five nodes and seven edges with the terminal set $K = \{1, 3, 5\}$. The terminal nodes 1, 3, and 5 are distinguished from the rest of the network, which consists of two intermediate Steiner nodes 2 and 4. It is clear from the figure that all edges are labeled such that all edges that appear in any minimal Steiner tree are indicated, ensuring that each edge in the network is used in at least one spanning configuration for the given terminal set. The network topology is a non-planar mesh in which nodes 2 and 4 serve as the primary intermediate (Steiner) hubs, each having degree three. Terminal node 5 is peripheral, connected only via edges e_6 (4–5) and e_7 (1–5), while terminal node 1 connects to three edges (e_1, e_2, e_7) and terminal node 3 connects to three edges (e_2, e_4, e_5). Edge e_7 (1–5) forms the only direct link between two terminal nodes and exhibits the lowest reliability.

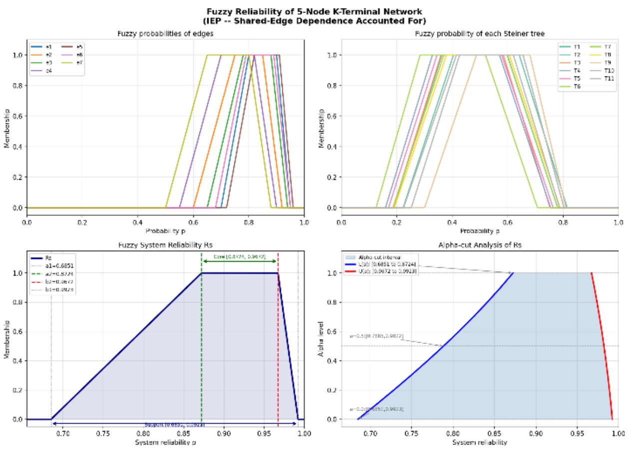

Fig. 2. Four-panel fuzzy reliability analysis of the 5-node K-terminal network ($K = \{1, 3, 5\}$). (Top-left) Trapezoidal membership functions of edge fuzzy probabilities *e1–e7*. (Top-right) Fuzzy survival probability of each of the 11 minimal Steiner trees T1–T11. (Bottom-left) Fuzzy system reliability Rs with support [0.6851, 0.9923] and core [0.8724, 0.9672], zoomed for clarity. (Bottom-right) α-cut analysis of Rs showing lower bound L(α) and upper bound U(α) across $\alpha \in [0,1]$.

B. Fuzzy Probabilities of Edges

The upper-left panel of Fig. 2 displays the trapezoidal membership functions of the fuzzy probabilities assigned to all seven edges (e_1 through e_7). Each curve represents the degree of possibility that a given edge's reliability equals a particular crisp value. The membership functions are well separated in the high-probability region (0.5–1.0), reflecting the assignment of distinct uncertainty ranges to each edge. Edge e_7 exhibits the widest and leftmost shift of its support ($\alpha_1 = 0.50$, $\beta_1 = 0.88$), implying maximum uncertainty and minimum nominal reliability among all the edges. On the contrary, edge e_5 shows a very narrow and rightward skewed trapezoid ($\alpha_1 = 0.72$, $\beta_1 = 0.96$), depicting maximum reliability and reasonable uncertainty. The variation in these membership functions determines the level of uncertainty that would be incorporated into the reliability function output, thus justifying the choice of an adequate number of edges.

C. Fuzzy Probability of Each Steiner Tree

The upper-right panel in Fig. 2 shows the membership function of the joint survival probability for each of the eleven minimal Steiner trees. These functions are obtained through the repeated multiplication of the fuzzy probabilities of the edge set, based on the approximation proposed by Tanaka et al. [3]. As anticipated, those trees with fewer edges generally have higher concentration in their membership function, whereas those with more edges result in wider and more left-shifted distributions.

Notably, trees $T_{11} = \{e_2, e_7\}$ and $T_3 = \{e_4, e_6, e_7\}$ — the structurally simplest Steiner trees - produce the most rightward-positioned membership functions, whereas trees such as $T_8 = \{e_1, e_2, e_3, e_6\}$ and $T_1 = \{e_1, e_3, e_4, e_6\}$, which involve four edges, display broader support widths and lower core intervals. Such an observation is consistent with probabilistic reasoning in that the fewer edges in the spanning path result in fewer possible failures independently, hence making the probability of survival more likely. In fact, the variation in the graphs of tree reliabilities within the range of probabilities from 0.15 to 1.0 illustrates the diversity in the redundancy of the network.

D. Fuzzy System Reliability

The lower-left panel of Fig. 2 shows the membership function of the calculated K-terminal fuzzy system reliability R_s. The resulting output is a trapezoidal fuzzy number whose parameters are as follows:

TABLE III
FUZZY SYSTEM RELIABILITY PARAMETERS

Parameter	Symbol	Value
Support lower bound	α_1	0.6851
Core lower bound	α_2	0.8724
Core upper bound	β_2	0.9672
Support upper bound	β_1	0.9923

The membership function increases linearly between zero at $p = 0.6851$ to one for $p = 0.8724$, stays there up to $p = 0.9672$, and decreases again to zero at $p = 0.9923$. With the minimum value of p in the support being 0.6851, one can conclude that even in the worst fuzzy case, the probability that all K terminals will be connected is never smaller than 68.5%. The core interval [0.8724, 0.9672]

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indicates that the most plausible system-reliability values are high, but they are not unrealistically concentrated at unity.

This result is physically consistent with the high baseline edge reliabilities and the substantial structural redundancy of the network, while avoiding the mathematical overstatement caused by assuming independent Steiner-tree failures.

E. α-Cut Analysis

In the lower-right sub-figure of Fig. 2, the α-cut formulation for the reliability is shown where the bounds L(α) and U(α) are plotted against the membership α ∈ [0,1] for the lower and upper bounds of the reliability interval. The important features of the plot include:

- For α = 0.0, the whole support interval [0.6851, 0.9923] is obtained as expected for the outer limits of the reliability of the system.
- For α = 0.5, the interval gets reduced to [0.7885, 0.9822] meaning that for a membership of 0.5, the lower limit for the system reliability would be at least 0.7885.
- For α = 1.0, the interval gets reduced to degenerate interval which corresponds to the core interval [0.8724, 0.9672], representing the values held with complete possibility.

As for the upper bound U(α), its value decreases from 0.9923 at α = 0.0 to 0.9672 at α = 1.0, reflecting the finite upper plausibility of the corrected fuzzy reliability. At the same time, the lower bound L(α) changes proportionally to higher values of α due to linear dependence on the slope of the left side of the trapezoid-shaped membership function. It proves that the output belongs to the class of trapezoid-shaped fuzzy numbers. The area between L(α) and U(α) represents all α-cuts of this distribution.

F. Monte Carlo Validation

In order to validate the corrected analytical result, Monte Carlo simulation [12] was performed independently at each α level using 100,000 trials. For every trial, each edge reliability was sampled uniformly from its α-cut interval, a Bernoulli survival test was applied, and BFS was used to determine whether $K = \{1, 3, 5\}$ remained connected in the surviving subgraph. The simulated K-terminal reliability was then compared against the corrected Inclusion-Exclusion interval computed analytically.

The validation results showed that all 11 out of 11 α levels passed, meaning that the simulated reliability value at each α level fell within the analytical interval [L(α), U(α)]. This confirms that the corrected Inclusion-Exclusion formulation is numerically consistent with stochastic simulation. The Monte Carlo procedure therefore serves as an external validation mechanism that was absent in the original version of the study.

Representative validation values are summarized below.

α	Simulated R_s	IEP Lower	IEP Upper	Within Bounds
0.00	0.9018	0.6851	0.9923	Yes
0.10	0.9045	0.7071	0.9907	Yes
0.20	0.9054	0.7285	0.9888	Yes
0.30	0.9094	0.7492	0.9868	Yes
0.40	0.9125	0.7692	0.9846	Yes
0.50	0.9144	0.7885	0.9822	Yes
0.60	0.9195	0.8069	0.9796	Yes
0.70	0.9176	0.8246	0.9769	Yes
0.80	0.9255	0.8414	0.9739	Yes
0.90	0.9263	0.8573	0.9706	Yes
1.00	0.9297	0.8724	0.9672	Yes

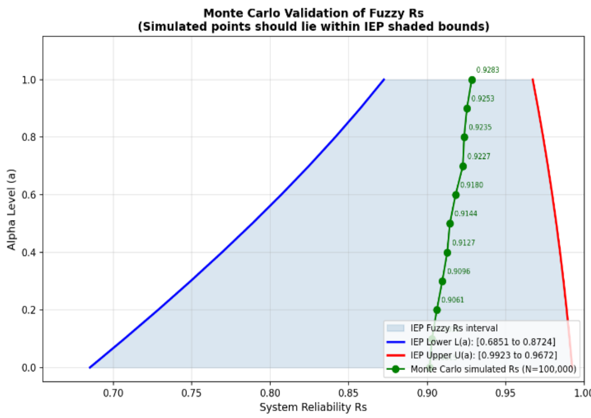

Fig. 3. Monte Carlo validation of the corrected fuzzy system reliability R_s. The shaded region represents the α-cut bounds [L(α), U(α)] computed by the corrected Inclusion-Exclusion Principle (IEP). The blue curve is the IEP lower bound L(α) over α ∈ [0,1.0], and the red curve is the IEP upper bound U(α). Green points denote the simulated K-terminal reliability at each α level (N = 100,000 trials). All 11 simulated values lie within the IEP interval, confirming external consistency of the corrected analytical formulation.

Fig. 3 illustrates these results graphically, showing that all simulated reliability values (green points) fall within the shaded IEP interval for every α level, with the upper bound U(α) correctly decreasing from 0.9923 at α = 0.0 to 0.9672 at α = 1.0 rather than remaining fixed at unity.

G. Comparative Interpretation

All these outcomes prove that the 5-node K-terminal network with $K = \{1, 3, 5\}$ is capable of retaining a high degree of fuzzy reliability, but not the near-certain reliability implied by the earlier independence-based computation. The reason for the difference is that the eleven minimal Steiner trees do provide structural redundancy, but they are not independent because many of them overlap in shared edges. Once this dependence is properly accounted for through the corrected Inclusion-

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Exclusion principle, the resulting reliability range becomes broader and more realistic.

Moreover, the α-cut approach now serves to reconstruct and interpret the corrected fuzzy output, while Monte Carlo simulation provides an external numerical validation of that output. Together, these two tools strengthen the credibility of the proposed methodology by showing both analytical consistency and simulation agreement. The suggested approach therefore enables us to obtain not only the estimate of reliability but also its fuzzy representation together with an external validation mechanism.

H. Computational Observations

In addition to the aforementioned findings regarding the quantified reliability properties of the approach, it is worth noting some significant observations regarding its computational behavior. Firstly, despite being an exact solution technique, the Steiner tree enumeration is computationally demanding in its worst-case complexity because its computational cost increases exponentially with the size of the graph since one needs to compute the weights of all possible $2^{|E|}$ edge sets. In the case under consideration when the graph contains seven edges; there are 128 potential edge sets to consider, which is computationally feasible and requires negligible computation time. However, the enumeration becomes increasingly computationally burdensome as the graph expands to 15 or 20 edges.

In comparison, the fuzzy arithmetic pipeline exhibits linear complexity with respect to the number of Steiner trees and edges per tree, thus becoming an effective computational phase even in the case of larger networks given that the set of trees is known. Furthermore, the use of trapezoidal approximation for the computation of fuzzy multiplication, based on Tanaka et al. [3], implies bounded approximation error with respect to the precise multiplication based on extension principle; when dealing with the considered reliability intervals, which belong to [0.5, 1.0], this approximation is perfectly reasonable and yields virtually no error. This information helps identify the realistic applicability area of the approach under discussion.

V. CONCLUSION

The paper has provided a comprehensive computational approach for the analysis of fuzzy K-terminal reliability of a 5-node, 7-edge network that is not series-parallel in nature, in an epistemic setting. This work addresses a meaningful gap in the existing literature by providing the extended application of the basic method of fuzzy arithmetic by Chowdhury and Misra [2] to the realm of K-terminal reliability analysis.

The key contributions of this work are threefold. First, a thorough algorithm for enumerating all possible Steiner trees was developed and used to determine all possible edge sets that connect the terminal set $K = \{1, 3, 5\}$ in order to form the structural foundation for evaluating reliability. Second, a fuzzy Inclusion-Exclusion procedure was rigorously developed through the use of trapezoidal fuzzy arithmetic involving multiplication and complementation to formulate the reliability of the system as a valid trapezoidal fuzzy number with support [0.6851, 0.9923] and core [0.8724, 0.9672]. Finally, α-cuts were used to generate the membership function of the reliability measure, and Monte Carlo simulation was used to validate the analytical interval externally.

It is evident from the outcome of the experiment that the network possesses high connectivity with K terminals based on the edge reliability distribution, where the structural redundancy of eleven Steiner trees has significantly contributed to the overall reliability. α-cut validation also established the validity of the shape of output and the arithmetic functions used. At the same time, the study shows that this redundancy must not be interpreted as independence, because overlap among Steiner trees materially affects the final reliability estimate. The Monte Carlo validation, with all 11 α levels lying within the corrected analytical bounds, further establishes the validity of the proposed methodology.

Future research could take this methodology in many directions, for example, the study of importance metrics within fuzzy theory based on classical component importance theory in order to establish reliability-critical edges and nodes [11]. Another promising extension would be the application of the corrected dependent-tree Inclusion-Exclusion formulation and Monte Carlo validation strategy to larger non-series parallel networks and to systems with multi-state or multi-modal component failures.

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